AN ANALYSIS OF THE TIPPING OF THE UPPER SECTION OF WTC 2

By

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1.0 Introduction

An examination of the many available videos of the collapse of WTC 2 shows that the block of floors above the aircraft impact zone did not simply descend vertically onto the floors below but tipped over as it fell. In Chapter 2 of the FEMA Report on the World Trade Center Disaster we read:

"*A review of aerial photography of the site, following the collapse, as well as identification of pieces of structural steel from WTC 2, strongly suggests that the top portion of the tower fell to the south and east, striking Liberty Street and the Bankers Trust building.*"

A discussion of the initiation of the asymmetric collapse of the upper section of WTC 2 is given in the NIST Final Report. Thus on page 308 of Chapter 9 of NISTSTAR 1-6 we read in reference to the condition of WTC 2 just moments before it began to collapse:

"*The entire section of the building above the impact zone began tilting as a rigid block (all four faces; not only the bowed and buckled east face) to the east (about 7° to 8°) and south (about 3° to 4°) as column instability progressed rapidly from the east wall along the adjacent north and south walls.*"

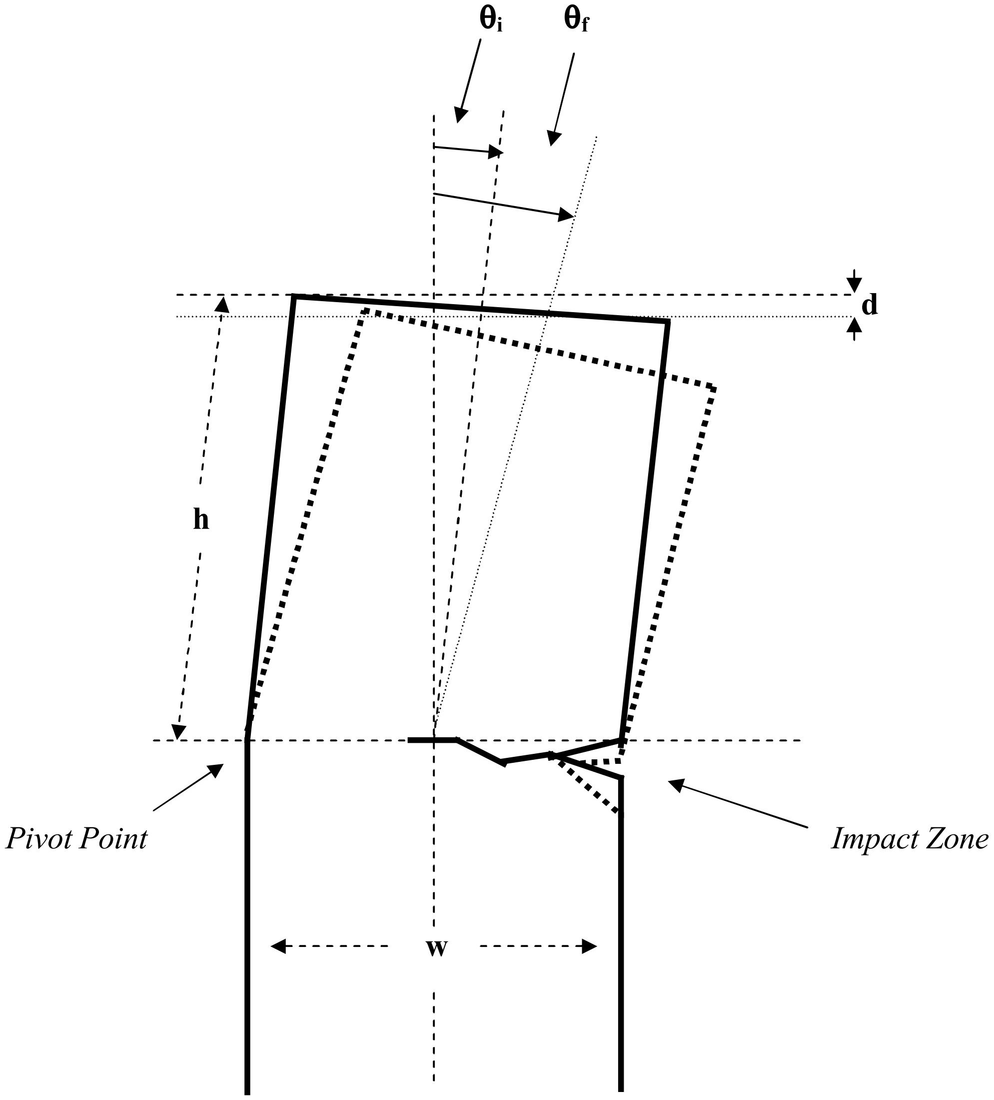
A recent article entitled: "*A Discussion of the Final NIST Report on the Collapse of WTC Buildings 1 and 2*", (available in pdf format on 911Myths.com), considers the tilting of WTC 2 prior to its collapse and notes that a 2° tilt of the upper section is the *largest* possible pre-collapse angle that is consistent with video and photographic observations of WTC 2. Thus, as the video evidence clearly shows, <u>the NIST Final Report greatly exaggerates the pre-collapse tilt angle of WTC 2</u>.

In this report we re-visit this issue by analyzing the *dynamics* of the rotational motion of the upper section of WTC 2 during the early stages of its collapse and show that the observed changes in the angular velocity are consistent with a <u>*small*</u> (less than 3°) initial angle of tilt.

2.0 Mathematics of the Rotation of the Upper Section of WTC 2

The rotation of the upper section of WTC 2 is clearly visible in the video and photographic record of 9-11 and was discussed by Bazant and Zhou, (B & Z), in one of the first studies of the collapse of WTC 1 & 2: "*Why did the World Trade Center Collapse?*" published in the September 2001 issue of Journal of Engineering Mechanics. Remarkably, however, *there appear to be no published reports of <u>the rate</u>* at which the tilting of the upper section of WTC 2 occurred. An Appendix to B & Z's paper does consider why the top part of WTC 2 tilted and assumes that this part of the building rotated as a rigid body about a pivot. We shall follow this approach using the collapse geometry shown in Figure 1

Figure 1: The Geometry of the Tipping of the Upper Section of WTC 2



h is the height of the block (= the distance from the impact zone to the top of the Tower)
w is the width of the Tower
$\boldsymbol{\theta_i}$ is the tilt angle at initial time $\mathbf{t_i}$
$\boldsymbol{\theta_f}$ is the tilt angle at final time $\mathbf{t_f}$

Thus we assume that at some initial time $\mathbf{t_i}$, the upper section of WTC 2 was already tilted from the local vertical by $\boldsymbol{\theta_i}$ degrees and subsequently moved, *solely under the action of gravity*, so that at time $\mathbf{t_f}$ it was tilted by $\boldsymbol{\theta_f}$ degrees. Newtonian mechanics requires that the initial rotational kinetic energy of the upper section, $\mathbf{T_i}$, is given by:

$$\mathbf{T_i = \tfrac{1}{2}\, I\, \{d\theta/dt\}_i^2}$$

Where **I** is the *moment of inertia* around the rotational axis. If the mass of the upper section of floors is represented by **M**, it is readily shown that:

$$\mathbf{I = 1/3\ M(\,h^2\ +\ w^2/4\,)}$$

where **h** and **w** are respectively the height and width of the upper section of WTC 2 as shown in Figure 1. Thus the initial rotational kinetic energy is:

$$\mathbf{T_i = 1/6\ M(\,h^2\ +\ w^2/4\,)\ \{d\theta/dt\}_i^2}$$

The initial *potential energy*, $\mathbf{P_i}$, of the upper section of WTC 2 is given by:

$$\mathbf{P_i = \tfrac{1}{2}\, Mgh \cos\theta_i}$$

where **g** is the acceleration due to gravity.

A similar set of equations may be written for the *final* kinetic and potential energies of the upper section of WTC 2 when it has tilted to angle $\boldsymbol{\theta_f}$ at time $\mathbf{t_f}$:

$$\mathbf{T_f = 1/6\ M(\,h^2\ +\ w^2/4\,)\ \{\,d\theta/dt\,\}_f^2} \quad \text{and} \quad \mathbf{P_f = \tfrac{1}{2}\, Mgh \cos\theta_f}$$

Energy conservation requires that:

$$\mathbf{T_i\ +\ P_i\ =\ T_f\ +\ P_f}$$

Substituting into this equation the appropriate energy expressions for the kinetic and potential energies in terms of the variables **M**, **h**, $\boldsymbol{\theta}$ and **t** and, for convenience, setting $\mathbf{\sqrt{2} \times b = \sqrt{[3gh\,/\,(\,h^2\ +\ w^2/4\,)]}}$, we arrive at the equation:

$$\mathbf{\{\,d\theta/dt\,\}_f^2\ =\ \{\,d\theta/dt\,\}_i^2\ + 2\, b^2\, \{\,\cos\theta_i\ -\ \cos\theta_f\,\}}$$

or,

$$\mathbf{\{\,d\theta/dt\,\}_f\ = \sqrt{[\,\{\,d\theta/dt\,\}_i^2\ + 2\, b^2\, \{\,\cos\theta_i\ -\ \cos\theta_f\,\}\,]}}$$

In order to simplify the solution of this equation for the calculation of **θ** as a function of **t** it is convenient to consider **$\{ d\theta/dt \}_i^2$** to be a constant we shall call **a^2**. In addition we consider the case where **θ_i** is a *small* angle so that **$\cos \theta_i$** is close to unity. By close to unity we mean to within 1 % remembering that cos 8° = 0.99

Hence we may write to a good approximation for initial angles ≤ 8°:

$$\{ d\theta/dt \}_f = \sqrt{[a^2 + 2 b^2 \{ 1 - \cos \theta_f \}]}$$

which upon integration yields,

$$\int d\theta_f / \sqrt{[a^2 + 2 b^2 \{ 1 - \cos \theta_f \}]} = \int dt$$

Evaluation of the integrand on the left hand side of this equation is facilitated by the additional approximation:

$$\cos x = 1 - x^2/2! + x^4/4! - \ldots$$

Hence,

$$1 - \cos \theta = 1 - (1 - \theta^2/2! + \theta^4/4! - \ldots$$

or,

$$1 - \cos \theta \approx ½ \theta^2$$

This approximation is accurate to 1 % for angles up to 20°. Hence we may write to a good approximation for angles ≤ 20°:

$$\int d\theta / \sqrt{[a^2 + b^2 \theta^2]} = \int dt$$

or,

$$\int d\theta / \sqrt{[a^2/ b^2 + \theta^2]} = b (t_f - t_i) = b\, t$$

It may be shown that:

$$\int dX / \sqrt{[A^2 + X^2]} = \ln \{ [X + \sqrt{(A^2 + X^2)}] / A \}$$

Hence by setting **$A = a/b$** (with **$a = (d\theta/dt)_i$** , and **$b = \sqrt{½ [3gh / (h^2 + w^2/4)]}$**), we may evaluate the above integral as follows:

$$t = 1/b \int d\theta / \sqrt{[a^2/ b^2 + \theta^2]} = 1/b \ln \{ \theta + \sqrt{[(d\theta/dt)_i^2 / b^2 + \theta^2]} / [(d\theta/dt)_i / b] \}$$

Now, since we have already stipulated that $\boldsymbol{\theta_i}$ is small, we will assume that $\mathbf{(d\theta/dt)_i^2}$ is also small and may be neglected. We may therefore write:

$$\mathbf{t = 1/b \ln \{ 2\theta / [(d\theta/dt)_i / b] \}}$$

<u>Evaluation of **b**</u>

For the upper section of WTC 2, **h** = 111 meters and **w** = 64 meters. With **g** = 9.8 m/s^2 we then have:

$$\mathbf{b = \sqrt{\{ (0.5 \times 3 \times 9.8 \times 111) / (111^2 + 64^2 / 4) \}} = 0.3497\ s^{-1}}.$$

It follows that the rotation of the upper section of WTC 2 is governed by the equation:

$$\underline{\text{t (in seconds)} = \mathbf{2.86 \ln \{ \theta / [1.43 (d\theta/dt)_i] \}}}$$

This equation shows that for a rigid structure of known height and width tipping over under the action of gravity, the time to rotate through an angle $\boldsymbol{\theta}$ depends on the log of the angle and the initial rate of rotation $\mathbf{(d\theta/dt)_i}$.

3.0 Observational Data Derived From Videos of the WTC 2 Collapse

In our recent article entitled: "*A Discussion of the Final NIST Report on the Collapse of WTC Buildings 1 and 2*", (available in pdf format on 911Myths.com), we presented data based on measurements from a number of videos for the tilt angle and drop distances during the first four seconds of the collapse of WTC 2. These data are reproduced in Table 1 below.

Table 1: WTC 2 Tilt Angles and Observed and Calculated Drop Distances

Time (s)	Tilt Angle (degrees)	Observed drop distance (m)	Calc. rotational drop distance (m)	True vertical drop distance (m)
0	-	-	0	0
0.5	-	1.9	1.1	0.8
1.0	2.5	4.0	2.4	1.6
1.5	4.5	11.0	5.2	5.8
2.0	6.9	17.9	8.5	9.4
2.5	10.2	25.5	13.1	12.4
3.0	14.7	33.6	19.9	13.7
3.5	19.4	44.5	27.6	16.9
4.0	25.2	56.9	37.8	19.1

In Section 2.0 of this report a mathematical model of the rotation of the upper section of WTC 2 was developed based on the requirement for conservation of rotational plus potential energy. We now compare our model with the observational data in Table 2 in order to verify that WTC 2 tipped over at a "natural" rate, i.e. at an angular velocity and acceleration that are consistent with the action of gravitational forces.

The tilt-angle vs. time data in Table 1 are plotted in Figure 2. To analyze these data we note that the equation given at the end of Section 2.0 for the tilting motion of a rigid structure under the action of gravity may be expressed as:

$$\boldsymbol{\theta = \theta_o \, e^{b\,t}}$$

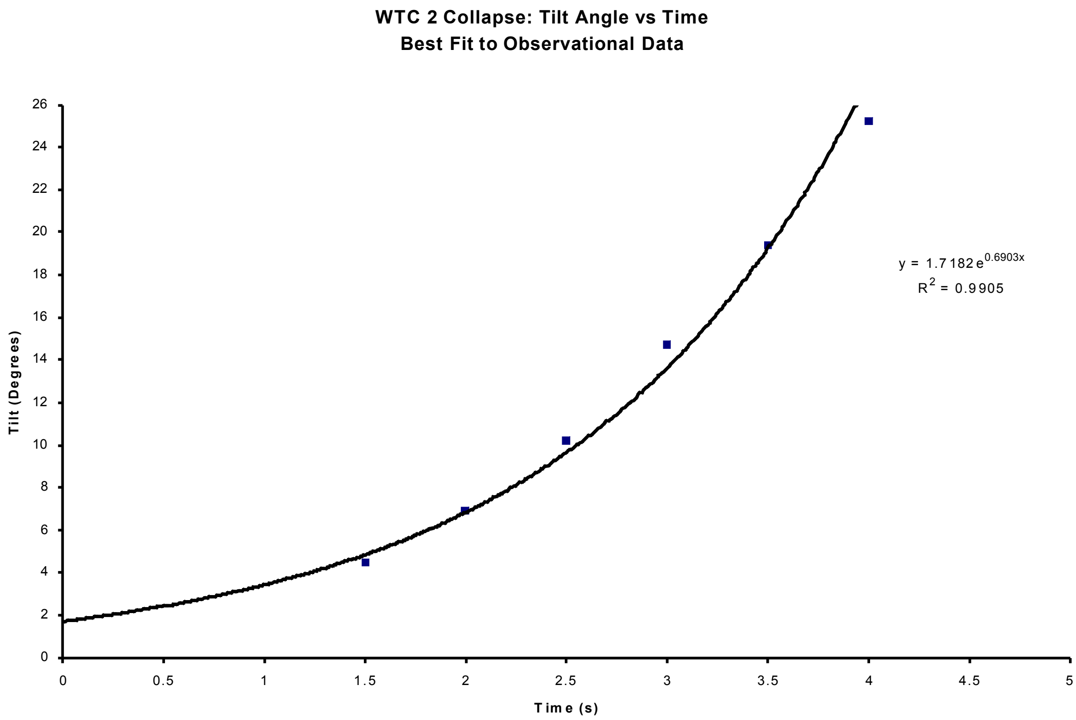
where,

$$\boldsymbol{\theta_o = (d\theta/dt)_i \,/\, 2b}$$

An equation of this form was fitted to the curve in Figure 2 with the following result:

$$\boldsymbol{\theta = 1.72\, e^{0.69\,t}} \;;\; \text{hence, } \boldsymbol{\theta_o = 1.72°,\ b = 0.69\ s^{-1}} \text{ and } \boldsymbol{(d\theta/dt)_i = 2.37°\ s^{-1}}$$

Figure 2: WTC 2 Collapse: Tilt Angle vs. Time Data and Curve Fit



4.0 Discussion

In this paper we have developed a mathematical model of the tipping of the upper section of WTC 2 during its collapse. We have also presented observational data for the tilt angle of WTC 2 as a function of time after collapse initiation based on measurements taken from videos (or still frames) of the demise of WTC 2. We now compare our mathematical model with the observational data.

The mathematical model developed in Section 2.0 leads to the equation:

$$\mathbf{t = 2.86 \ln \{ \theta / [1.43 (d\theta/dt)_i] \}}$$

By rearranging this equation we arrive at an expression for the *theoretical* tilt angle, $\boldsymbol{\theta_{the}}$, as a function of time, **t**:

$$\boldsymbol{\theta_{the} = 1.43 (d\theta/dt)_i\, e^{0.35 t}}$$

Here, $\boldsymbol{(d\theta/dt)_i}$ is the *initial* rate of tilt, or angular velocity, of the upper section of WTC 2 - an unspecified quantity in the theoretical model.

By comparison, the equivalent expression for the *observed* tilt angle, $\boldsymbol{\theta_{obs}}$, derived in Section 3.0 is:

$$\boldsymbol{\theta_{obs} = 0.72 (d\theta/dt)_i\ e^{0.69 t}}$$

The observational data show that:

$$\boldsymbol{(d\theta/dt)_i = 2.37° s^{-1}}$$

Hence, remembering that,

$$\boldsymbol{\theta = \theta_o e^{b t}} \text{ and } \boldsymbol{\theta_o = (d\theta/dt)_i / 2b},$$

the pre-exponential factor, $\boldsymbol{\theta_o}$, may be determined from $\boldsymbol{\theta_o = 2.37 \times 0.72 = 1.72°}$

The difference in the exponential coefficient, **b**, (0.35 s^{-1} for the theoretical case and 0.69 s^{-1} for the observational case), is apparent in the above expressions for $\boldsymbol{\theta_{the}}$ and $\boldsymbol{\theta_{obs}}$. This difference may be explained by first noting that $\mathbf{b \approx \sqrt{[3g/2h]}}$ and thus depends inversely on $\mathbf{\sqrt{h}}$, where **h** is the height of the upper section of WTC 2. Many videos clearly show that the upper section of WTC 2 *was crushed* as it collapsed. This loss of rigidity of the upper section effectively reduced **h** from its initial value of (30 × 3.7) or 111 meters to something significantly smaller by the time the block had rotated 25° or so. Hence it is not surprising that the *observed* value of **b** is <u>larger</u> than predicted for h = 111 meters, (see below).

Evaluation of the Pre-exponential Coefficient, $\boldsymbol{\theta_o}$, and the Initial Rate of Rotation $\mathbf{(d\theta/dt)_i}$

In a strictly formal sense $\boldsymbol{\theta_o}$ represents the tilt angle of the upper section of WTC 2 at time $\mathbf{t = 0}$. This could, at first sight, be defined as the time of collapse initiation. However, because of the relationship,

$$\boldsymbol{\theta_o = (d\theta/dt)_i / 2b},$$

we see that $\mathbf{(d\theta/dt)_i}$ is not *necessarily* equal to zero at $\mathbf{t = 0}$. In fact, if $\mathbf{(d\theta/dt)_i}$ was equal to zero at time zero, the upper section of WTC 2 would not rotate at all.

This situation arises because our mathematical model describes the rotation of a body *moving* between an initial and final state under the action of gravity. The model says nothing about the forces that set the body in motion, such as an initial "push"; and neither does the model consider forces that may "resist" the rotation.

To establish a theoretical basis for the observation that $\mathbf{(d\theta/dt)_i} = 2.37° \text{ s}^{-1}$, we note from Figure 1 that the "drop distance" of the right-hand side of our representation of the WTC 2 "leaning tower" is given by:

$$\mathbf{d = w \sin \theta}$$

In our previous article: "*A Discussion of the Final NIST Report on the Collapse of WTC Buildings 1 and 2*", it was shown that *prior to collapse initiation* the southeast corner of WTC 2 tilted down relative to its undamaged state by as much as 1 meter. From the above equation, with $\mathbf{w}$ = 64 meters, we conclude that during this pre-collapse phase the tilt angle of the upper section of WTC 2 was already about 0.9°.

WTC 2 started its rapid, "near free fall", collapse about 56 minutes after it was struck by an aircraft, and during this post-impact, pre-collapse period the intense fires within the building gradually weakened the structure and induced a slow but irrevocable buckling of columns near the impact zone. Thus we could say that WTC 2 actually started to tip over *as soon as it was hit by an aircraft*, but the tipping was not noticeable because the average angular velocity was only about 1° per hour or $0.0003° \text{ s}^{-1}$.

At *one minute* before collapse the angular velocity of the upper section of WTC 2 would have increased to about $0.03° \text{ s}^{-1}$. However, even this rate of tipping was too small to be noticeable. Nonetheless, the necessary conditions for the total collapse of WTC 2 were now being established through the continuous build up of strain energy in the columns on the south and east sides of the Tower. Eventually the lowering of one side of the building was such that the maximum strain energy capacity of the structure was reached. At this critical juncture the columns on the weakened sides of the building underwent rapid failure and the 30 floors above the impact zone pivoted *en masse* about the northeast corner, causing this section of the Tower to fall obliquely on to the floor below.

Because this failure occurred when the floors in the southeast corner of the 80th floor of WTC 2 were already lowered by up to 1 meter from the initial 3.7 m floor spacing, the upper block would have pivoted a maximum distance of 2.7 m before impacting the floor below. Application of the formula $\mathbf{d = w \sin \theta}$ shows that such a failure mechanism would increase the initial 0.9° tilt angle of the upper section of WTC 2 by a further 2.4° giving a tilt angle at the moment of impact of the upper block of floors with the floor below of 3.3°. Thus for tilt angles between 0.9° and 3.3° the tipping of the upper section was occurring under the action of gravity and the conditions for the application of our mathematical description of the rotational motion were fully satisfied.

We conclude that $\boldsymbol{\theta_o}$ should be in the range $3.3° \geq \boldsymbol{\theta_o} \geq 0.9°$, consistent with the observed value of 1.7° Furthermore, because of the relation, $\boldsymbol{\theta_o = (d\theta/dt)_i / 2b}$, with **b** equal to 0.35 s^{-1}, (its value *before* the upper section was crushed), we also conclude that $2.3° s^{-1} \geq \boldsymbol{(d\theta/dt)_i} \geq 0.6° s^{-1}$. This is close to, but a little below, the observed value of $2.37° s^{-1}$

Bazant and Zhou, (B & Z), in their paper: "*Why did the World Trade Center Collapse?*" show that the horizontal shear force at the base of the tilting upper section of WTC 2 is equal to $\mathbf{3/8Mg \sin 2\theta}$ and estimate that the plastic shear resistance of WTC 2 columns near the impact zone was about 1/10.3 of the *maximum* horizontal shear. From this estimate B & Z conclude that the horizontal shear force equaled the plastic limit of the affected columns when $\mathbf{\sin 2\theta = 1/10.3}$ or when $\boldsymbol{\theta = 2.8°}$. Thus B & Z predict that the upper section of WTC 2 should become unstable at a tilt angle of about 3°, in good agreement with our mathematical model.

5.0 Summary and Conclusions

The collapse of WTC 2 began with a tilting or *rotational* motion of the upper section of the Tower about a "hinge" at the 80th floor. This rotational motion, which commenced at a tilt angle ≈ 2°, was caused by an almost instantaneous multi-column failure that eliminated the structural support on one side of WTC 2 near the impact zone. Once set in motion, the upper block moved with a nearly "free" rotational trajectory of a body pivoting under the constant force of gravity. This behavior was sustained at tilt angles up to about 25°. Thereafter the motion of the block changed somewhat although the suggestion that the tilting suddenly stopped is *not* correct.

What appears to happen is that the tilting upper section was continuously crushed near the 80th floor by its own momentum so that the rotation was no longer that of a *rigid body*. Eventually the "hinge" at the northeast corner failed and the descending block took on a more vertical motion. Interestingly, once the hinge failed, and the pivot became frictionless, the motion of the center of gravity is predicted to become vertical, causing a shift in the rotational axis. Unfortunately, however, details of this stage of the WTC 2 collapse were obscured by smoke, dust and flying debris.

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